Geometry of Quantum States

2008 J. Phys. A: Math. Theor. 41 019001

(http://iopscience.iop.org/1751-8121/41/1/019001)

J. Phys. A: Math. Theor. **41** (2008) 019001 (2pp) doi:10.1088/0305-4470/41/1/019001

Book reviews

Geometry of Quantum States
Ingemar Bengtsson and Karol Zyczkowski
2007 Cambridge University Press
418pp US$95.00, £50.00 (hardback)
ISBN: 978-0-5218-1451-5

A geometric framework for quantum mechanics arose during the mid 1970s when authors such as Cantoni explored the notion of generalized transition probabilities, and Kibble promoted the idea that the space of pure quantum states provides a natural quantum mechanical analogue for classical phase space. This central idea can be seen easily since the projection of Schrödinger's equation from a Hilbert space into the space of pure spaces is a set of Hamilton's equations. Over the intervening years considerable work has been carried out by a variety of authors and a mature description of quantum mechanics in geometric terms has emerged with many applications. This current offering would seem ideally placed to review the last thirty years of progress and relate this to the most recent work in quantum entanglement.

Bengtsson and Życzkowski's beautifully illustrated volume, *Geometry of Quantum States* (referred to as *GQS* from now on) attempts to cover considerable ground in its 418 pages. Its topics range from colour theory in Chapter 1 to quantum entanglement in Chapter 15—to say that this is a whirlwind tour is, perhaps, no understatement. The use of the work 'introduction' in the subtitle of *GQS*, might suggest to the reader that this work be viewed as a textbook and I think that this interpretation would be incorrect. The authors have chosen to present a survey of different topics with the specific aim to introduce entanglement in geometric terms—the book is not intended as a pedagogical introduction to the geometric approach to quantum mechanics. Each of the fifteen chapters is a short, and mostly self-contained, essay on a particular aspect or application of geometry in the context of quantum mechanics with entanglement being addressed specifically in the final chapter.

The chapters fall into three classifications: those concerned with the mathematical background, those which discuss quantum theory and the foundational aspects of the geometric framework, and applications of the geometric approach.

The first four chapters contain the standard mathematics required to understand the rest of the material presented: specific areas in colour theory, set theory, probability theory, differential geometry and projective geometry are all covered with an eye to the material that follows. Chapter 5 starts the first real discussion of quantum theory in *GQS* and serves as an elegant, succinct introduction to the geometry which underlies quantum theory. This may be the most worthwhile chapter for the casual reader who wants to understand the key ideas in this field.

Chapter 6 builds on the discussion in Chapter 5, introducing a group theoretic approach to understand coherent states and Chapter 7 describes a geometric tool in the form of an approach to complex projective geometry called 'the stellar representation'. Chapter 8 returns to a more purely quantum mechanical discussion as the authors turn to study the space of density matrices. This chapter completes the discussion which started in Chapter 5.

Chapter 9 begins the part of the book concerned with applications of the geometric approach. From this point on the book aims, specifically, to prepare the reader for the material in Chapter 15 beginning with a discussion on the purification of mixed quantum states. In the succeeding chapters a definite choice has been made to present a geometric approach to certain quantum information problems. For example, Chapter 10 contains an extremely well formulated discussion of measurement and positive operator-valued measures with several well illustrated examples and Chapter 11 reopens the discussion of density matrices. Entropy and majorization are again revisited in Chapter 12 in much greater detail than in previous chapters. Chapters 13 and 14 concern themselves with a discussion of various metrics and their relation to the problem of distinguishing between probability distributions and their suitability as probability measures.

D W Hook
Blackett Laboratory, Imperial College of Science Technology and Medicine, University of London, Prince Consort Road, London, SW7 2BW, UK

doi:10.1088/0305-4470/41/1/019001

Fearful Symmetry: The Search for Beauty in Modern Physics
A Zee
2007 Princeton University Press
376pp US$19.95, £11.95 (paperback)
ISBN 978-0-691-13482-6

It is easy to see beauty in symmetry when we look at buildings like the Taj Mahal or natural objects such as snowflakes; it is much harder to explain to a non-expert the beauty of equations or of symmetry concepts in relativity or in particle theory.

Tony Zee achieves this in a remarkable way, while he also manages to make many complicated concepts accessible to a reader who is genuinely interested and who has some basic/school knowledge of physics. To do this he invents various ordinary world analogies and exploits them in a masterful way. I liked, in particular, his analogy for the colour of quarks and the associated SU(3) symmetry provided by adding colour to ice-cream, which does not change its cost.

Of course, sometimes real beauty is associated with a small breakdown of symmetry. We are all familiar with this in music or in art. Tony shows that such a breakdown also has a role in physics and that it is often associated with unexpected and very deep and important concepts (parity, CP violation or baryon assymetry).

The book is an amazing achievement; although the main focus is on symmetry and beauty the author manages to explain most of the new and relevant concepts of modern physics, from quantum mechanics and relativity to superstrings and superbranes. And he does this with no equations and almost no mathematical symbols.

So who is this book intended for? Who will enjoy reading it? Clearly, it will be appreciated by all theoretical physicists, who probably will be primarily impressed by the way the book makes accessible so many very difficult concepts. I was particularly struck by Tony's ability to explain things in simple terms and to find relevant analogies. It will also be greatly enjoyed by the non-specialist but 'interested' reader; (s)he may find some concepts hard to follow but (s)he will get the general gist of the arguments. It will also be enjoyed by sixth-formers studying physics and quite possibly will attract some of them to study science at university. And as such it may help to stop or even reverse the general move away from hard sciences.

I think this book is of great quality and I would recommend it to almost everybody.

W J Zakrzewski
Department of Mathematical Sciences Science Laboratory, Durham University, South Road, Durham, DH1 3LE, UK

doi:10.1088/0305-4470/41/1/019002